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RADIATIVE RARE KAON DECAYS[1]

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Abstract

We review some recent theoretical results on radiative rare kaon decays. Particular attention is devoted to the channels $K \to \pi l \bar{l}$ and $K \to \pi\pi\gamma$, where we are (or might be) able to extract the short distance contributions. This is achieved by a careful study of the long distance part. We study also CP violating observables, which are sensitive also to extensions of the SM. As byproduct, we discuss interesting chiral tests.

1 Introduction

Rare Kaon decays are a very interesting place to test the Standard Model (SM) and its extensions [1, 2]. $K \to \pi\nu\overline{\nu}$ decays have the advantage not to be affected by long distance uncertainties and thus they are definetely very appealing [1, 2] . Here we study the complementary channels $K \to \pi\ell^+\ell^-$ and $K \to \pi\pi\gamma$ that can be studied either as byproduct of the previous ones or also as an independent search. The long distance contributions are in general not negligible and must be carefully studied in order to pin down the short distance part. The advantage is that these channels are more accessible experimentally.

2 $K_L \to \pi^0\ell^+\ell^-$, $K_S \to \pi^0\ell^+\ell^-$ and $K^+ \to \pi^+\ell^+\ell^-$

The effective current⊗current structure of weak interactions constrains short distance contribution to $K_L \to \pi^0\ell^+\ell^-$, analogously to $K_L \to \pi^0\nu\overline{\nu}$, to be direct CP violating [3]. However differentely from the neutrino case $K_L \to \pi^0\ell^+\ell^-$ receives also non-negligible long distance contributions: i) indirect CP violating from $1\gamma-$ exchange and ii) CP conserving from 2γ-exchange. Furthermore we must warn about the danger of the potentially large background

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contribution from $K_L \to e^+e^-\gamma\gamma$ to $K_L \to \pi^0 e^+e^-$ [4]. The present bounds from KTeV [5] are

$$B(K_L \to \pi^0 e^+e^-) < 5.6 \times 10^{-10} \quad B(K_L \to \pi^0 \mu^+\mu^-) < 3.4 \times 10^{-10}. \tag{1}$$

2.1 Direct CP violating contributions

Box+Z-penguin top loop induce the direct CP violating $K_L \to \pi^0 \ell^+\ell^-$ amplitude. QCD corrections have been evaluated at next-to-leading order [6] leading to the prediction

$$B(K_L \to \pi^0 e^+e^-)^{SM}_{CPV-dir} = 0.69 \times 10^{-10} \left[\frac{\bar{m}_t(m_t)}{170GeV}\right]^2 \left[\frac{\Im m(\lambda_t)}{\lambda^5}\right]^2;$$

where $\lambda_q = V^*_{qs}V_{qd}$ and using the present constrains on $\Im m(\lambda_t)$ one obtains the range [1, 6]

$$2.8 \times 10^{-12} \leq B(K_L \to \pi^0 e^+e^-)^{SM}_{CPV-dir} \leq 6.5 \times 10^{-12}.$$

Lately it has been pointed out the possibility of new physics to substantially enhance the SM predictions through effects that could be parametrized by an effective dimension-4 operator $\overline{s}dZ$ vertex Z_{ds} [7]; the CP violating contribution $\Im m(Z_{ds})$ and consequentely $K_L \to \pi^0\nu\overline{\nu}$ is constrained by the value of ε'/ε, while $\Re e(Z_{ds})$ and $K^\pm \to \pi^\pm\nu\overline{\nu}$ are limited by $K_L \to \mu\overline{\mu}$ [8, 9]. Also the recent large value of ε'/ε [10, 11] allows new sources of CP violating contributions [12, 13]. In particular, a large value for the Wilson coefficient C^-_g of the dimension-5 operators of the $|\Delta S| = 1$ effective hamiltonian

$$\mathcal{H}^{|\Delta S|=1;d=5}_{eff} = \left[C^+_\gamma Q^+_\gamma + C^-_\gamma Q^-_\gamma + C^+_g Q^+_g + C^-_g Q^-_g + \text{h.c.}\right], \tag{2}$$

where

$$Q^\pm_\gamma = \frac{Q_d e}{16\pi^2}(\bar{s}_L\sigma_{\mu\nu}d_R \pm \bar{s}_R\sigma_{\mu\nu}d_L)\cdot F^{\mu\nu} \tag{3}$$

and

$$Q^\pm_g = \frac{g}{16\pi^2}(\bar{s}_L\sigma_{\mu\nu}t^aG^{\mu\nu}_a d_R \pm \bar{s}_R\sigma_{\mu\nu}t^aG^{\mu\nu}_a d_L) \tag{4}$$

has been advocated for the large value of ε'/ε [14]. Indeed the SM allows only small values (suppressed by $s, d-$quark masses) for $C^\pm_\gamma$ and $C^\pm_g$. One can then check the consequences for $K \to \pi\ell\overline{\ell}$ and in general rare kaon decays of New Physics (NP) valus for all the Wilson coefficient in (2). Indeed one finds that $B(K \to \pi\ell\overline{\ell})_{NP}$ can be as much as one order of magnitude larger than the Standard Model prediction [15]. Lately it has been pointed out that C^+_g in (2) contributes also to the charge asymmetry in $K^+ \to 3\pi$ [16].

2.2 Indirect CP violating contribution, $K_S \to \pi^0 e^+e^-$ and $K^\pm \to \pi^\pm l^+l^-$

Short distance contributions to $K \to \pi \overset{*}{\gamma}$ are very small compared to long distance contributions evaluated then in Chiral Perturbation Theory (χPT) [17]. $K \to \pi \overset{*}{\gamma}$ ($K^\pm \to \pi^\pm \overset{*}{\gamma}$ and

$K_S \to \pi^0 \overset{*}{\gamma}$) decays start at $\mathcal{O}(p^4)$ in χPT with loops (dominated by the $\pi\pi-$cut) and counterterm contributions [18]. Higher order contributions ($\mathcal{O}(p^6)$) might be large, but are not completely under control since new (and with unknown coefficients) counterterm structures appear [19]. In Ref. [20] we have parameterized the $K \to \pi \overset{*}{\gamma}(q)$ form factor as

$$W_i(z) = G_F M_K^2 (a_i + b_i z) + W_i^{\pi\pi}(z) , \qquad i = \pm, S \tag{5}$$

with $z = q^2/M_K^2$, and where $W_i^{\pi\pi}(z)$ is the loop contribution, given by the $K \to \pi\pi\pi$ unitarity cut and completely known up to $\mathcal{O}(p^6)$. All our results in that reference are expressed in terms of the unknown parameters a_i and b_i, expected of $\mathcal{O}(1)$. At the first non-trivial order, $\mathcal{O}(p^4)$, $b_i = 0$, while a_i receive counterterm contributions not determined yet. At $\mathcal{O}(p^6)$, $b_i \neq 0$, while a_i receive new and unknown contributions. Due to the generality of (5), we expect that $W_i(z)$ is a good approximation to the complete form factor. From the $K^+ \to \pi^+ e^+ e^-$ experimental width and slope, E865 obtains [21]

$$a_+ = -0.587 \pm 0.010 \qquad b_+ = -0.655 \pm 0044 \tag{6}$$

Also the fit with (5), i.e. with the genuine chiral contribution $W_+^{\pi\pi}(z)$, is better ($\chi^2 /d.o.f. \sim 13.3/18$) than just a linear slope ($\chi^2/d.o.f. \sim 22.9/18$), showing the validity of the chiral expansion.Then the universality of the form factor in (5) is further tested by using (6) to predict the branching $B(K^+ \to \pi^+\mu^+\mu^-)$, which indeed perfectely agrees with the new experimental value by E865 $B(K^+ \to \pi^+\mu^+\mu^-)_{\rm exp} = (9.22 \pm 0.60 \pm 0.49) \cdot 10^{-8}$[22, 23]. This value is however larger [20] by 3.3 σ 's than the old experimental result [24]. Also the slope in the muon channel, though with large statistical errors is now consistent with (6) [22].

We should stress that it is not clear at the moment the meaning of the apparent slow convergence in the chiral expansion in $K^+ \to \pi^+ l^+ l^-$, indeed the values in (6) do not respect the naive chiral dimensional analysis expectation $b_+/a_+ \sim M_K^2/M_V^2$.

There is no model independent relation among a_S and a_+ and thus a secure determination of $B(K_L \to \pi^0 e^+ e^-)_{CP-indirect}$ requires a direct measurement of $B(K_S \to \pi^0 e^+ e^-)$, possibly to be performed by KLOE at DAΦNE [20]. The dependence from b_S is very mild and thus we predict $B(K_S \to \pi^0 e^+ e^-) \simeq 5.2\, a_S^2 \times 10^{-9}$. If we include the interference term among direct and indirect the CP –violating terms we obtain [20]

$$B(K_L \to \pi^0 e^+ e^-)_{CPV} = \left[15.3\, a_S^2 - 6.8 \frac{\Im m \lambda_t}{10^{-4}} a_S + 2.8 \left(\frac{\Im m \lambda_t}{10^{-4}} \right)^2 \right] \times 10^{-12} . \tag{7}$$

A very interesting scenario emerges for $a_S \lesssim -0.5$ or $a_S \gtrsim 1.0$. Since $\Im m\lambda_t$ is expected to be $\sim 10^{-4}$, one would have $B(K_L \to \pi^0 e^+ e^-)_{CPV} \gtrsim 10^{-11}$ in this case. Moreover, the $K_S \to \pi^0 e^+ e^-$ branching ratio would be large enough to allow a direct determination of $|a_S|$. Thus, from the interference term in (7) one could perform an independent measurement of $\Im m\lambda_t$, with a precision increasing with the value of $|a_S|$.

2.3 CP conserving contributions: "$\gamma\gamma$" intermediate state contributions

The general amplitude for $K_L(p) \to \pi^0 \gamma(q_1) \gamma(q_2)$ can be written in terms of two independent Lorentz and gauge invariant amplitudes $A(z, y)$ and $B(z, y)$:

$$M^{\mu\nu} \;=\; \frac{A(z,y)}{m_K^2}\,(q_2^\mu\, q_1^\nu \,-\, q_1\cdot q_2 g^{\mu\nu}) \;+\; \frac{2\,B(z,y)}{m_K^4}\,(-p\cdot q_1\, p\cdot q_2\, g^{\mu\nu} \,- \\ -\, q_1\cdot q_2\, p^\mu p^\nu \,+\, p\cdot q_1\, q_2^\mu p^\nu \,+\, p\cdot q_2\, p^\mu q_1^\nu\ ) \tag{8}$$

where $y = p\cdot(q_1 - q_2)/m_K^2$ and $z \;=\; (q_1+q_2)^2/m_K^2$. Then the double differential rate is given by

$$\frac{\partial^2\Gamma}{\partial y\,\partial z} = \frac{m_K}{2^9\pi^3}[\,z^2\,|\,A\,+\,B\,|^2 + \left(y^2 - \frac{\lambda(1,r_\pi^2,z)}{4}\right)^2 \,|\,B\,|^2\,]\ , \tag{9}$$

where $\lambda(a,b,c)$ is the usual kinematical function and $r_\pi = m_\pi/m_K$. Thus in the region of small z (collinear photons) the B amplitude is dominant and can be determined separately from the A amplitude. This feature is crucial in order to disentangle the CP-conserving contribution $K_L \to \pi^0 e^+e^-$. The $\gamma\gamma$ intermediate state can be i) real or ii) virtual, generating respectively an absorptive (two-photon discontinuity) and dispersive contribution to $K_L \to \pi^0 e^+e^-$. It has been shown in a model that i) is dominant [19], and further support might come from the experimental[25] and theoretical [26] study of $K_L \to \pi^0 e^+e^-\gamma$.

The two photons in the A-type amplitude are in a state of total angular momentum $J = 0$ (J, total diphoton angular momentum), and it turns out that for this contribution $A(K_L \to \pi^0 e^+e^-)_{J=0} \sim m_e$ (m_e electron mass) [27]; however the higher angular momentum state B-type amplitude in (8), though chirally and kinematically suppressed for $A(K_L \to \pi^0\gamma\gamma)$, generate $A(K_L \to \pi^0 e^+e^-)_{J\neq 0}$ competitive with the CP violating contributions [19].

The leading finite $\mathcal{O}(p^4)$ amplitudes of $K_L \to \pi^0\gamma\gamma$ generates only the A–type amplitude in Eq. (9) [28]. This underestimates the observed branching ratio, $(1.68\pm0.07\pm0.08)\times10^{-6}$ [29] by a large factor but reproduces the experimental spectrum, predicting no events at small z. The two presumably large $\mathcal{O}(p^6)$ contributions have been studied: i) the $\mathcal{O}(p^6)$ unitarity corrections [30, 31, 32] that enhance the $\mathcal{O}(p^4)$ branching ratio by 40% and generate a B –type amplitude, ii) the vector meson exchange contributions that are in general model dependent [33, 34] but can be parameterized $K_L \to \pi^0\gamma\gamma$ by an effective vector coupling a_V [34] . Then the contribution to $K_L \to \pi^0 e^+e^-$ is determined by the value of a_V. The agreement with experimental $K_L \to \pi^0\gamma\gamma$ rate and spectrum would demand $a_V \sim -0.8$ [31, 35].

It would be desirable to have a theoretical understanding of this value. Indeed we have related a_V with the $K_L \to \gamma\gamma^*$ linear slope, b [35]; the experimental value is $b_{exp} \;=\; 0.81 \pm 0.18$. Theoretically the slope b is also generated by vector meson exchange contribution.

We can evaluate now a_V and the $K_L \to \gamma\gamma^*$ slope b in factorization (FM), i.e. writing a *current*$\times$ *current* structure

$$\mathcal{L}_{FM} \;=\; 4\,k_F\,G_8\,\langle\,\lambda\,J_\mu\,J^\mu\,\rangle \;+\; h.c. \quad , \tag{10}$$

where $\lambda \,\equiv\, \frac{1}{2}\,(\lambda_6 \,-\, i\lambda_7)$, G_8 is determined from $K \to \pi\pi$ and the fudge factor $k_F \sim \mathcal{O}(1)$ has to be determined phenomenogically. A satisfactory understanding of the model would require $k_F \sim 0.2 - 0.3$, to match the perturbative result.

There are two ways to derive the FM weak lagrangian generated by resonance exchange (this corresponds to different ways to determine the conserved current J_μ) [35, 36] :

($\mathcal{A}$) To evaluate the strong action generated by resonance exchange, and then perform the factorization procedure in Eq. (10). By this way, since we apply the FM procedure once the vectors have already been integrated out the lagrangian is generated at the kaon mass scale.

($\mathcal{B}$) Otherwise, we can first write down the spin–1 strong and weak chiral lagrangian. The weak resonance coupling constants are determined in factorization. We then integrate out the resonance fields so that the effective lagrangian is generated at the scale of the resonance.

In principle the two effective actions do not coincide and phenomenology may prefer one pattern [36]. In the case at hand, $\mathcal{A}$ and $\mathcal{B}$ give different structures, however they both generate a good phenomenology with one free parameter k_F , i.e.

$$a_V \simeq -0.72 \quad , \quad b \simeq 0.8 - 0.9 \, , \tag{11}$$

but with different value of k_F : $\mathcal{A} \Rightarrow k_F = 1$ while $\mathcal{B} \Rightarrow k_F = 0.2$. Interestingly this seems to suggest that the matching should be performed at the resonance scale.

Very interestingly the new data from KTeV [29] confirms sharply our prediction for a_V : $a_V = -0.72 \pm 0.05 \pm 0.06$ and show a clear evidence of events at low z. This turns in a stringent limit for the CP conserving contribution to $K_L \to \pi^0 e^+ e^-$: $1. < B(K_L \to \pi^0 e^+ e^-) \cdot 10^{12} < 4$ [19, 35]. The direct measurement of the events at low z will give a direct, model independent and precise determination of the CP conserving contribution to $K_L \to \pi^0 e^+ e^-$.

3 $K \to \pi\pi\gamma$

The $K(p) \to \pi(p_1)\pi(p_2)\gamma(q)$ amplitude is usually decomposed also in electric (E) and the magnetic (M) terms [2]. Defining the dimensionless amplitudes E and M as in [37, 38, 2], we can write:

$$A(K \to \pi\pi\gamma) = \varepsilon_\mu(q) \left[E(z_i)(p_1 \cdot q \, p_2^\mu - p_2 \cdot q \, p_1^\mu) + M(z_i) \epsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} q_\sigma \right] / m_K^3, \tag{12}$$

where

$$z_i = \frac{p_i \cdot q}{m_K^2}, \qquad \text{and} \qquad z_3 = \frac{p_K \cdot q}{m_K^2} = z_1 + z_2. \tag{13}$$

In the electric transitions one generally separates the bremsstrahlung amplitude E_B : if eQ_i is the electric charge of the pion π

$$E_B(z_i) \doteq \frac{eA(K \to \pi_1\pi_2)}{M_K z_3} \left(\frac{Q_2}{z_2} - \frac{Q_1}{z_1} \right). \tag{14}$$

E_B is generally enhanced due to the factor $1/E_\gamma^*$ for $E_\gamma^* \to 0$, where E_γ^* is the photon energy in the kaon rest frame. Summing over photon helicities there is no interference among electric and magnetic terms:

$$\begin{aligned} \frac{\mathrm{d}\Gamma}{\mathrm{d}z_1 \mathrm{d}z_2} &= \frac{M_K}{4(4\pi)^3} \left(|E(z_i)|^2 + |M(z_i)|^2 \right) \\ &\times \left[z_1 z_2 (1 - 2z_3 - r_1^2 - r_2^2) - r_1^2 z_2^2 - r_2^2 z_1^2 \right], \end{aligned} \tag{15}$$

($r_m = m_\pi/m_K$). At the lowest order (p^2) in χPT one obtains only E_B

Magnetic and electric direct emission amplitudes, appearing at $\mathcal{O}(p^4)$, can be decomposed in a multipole expansion (see Ref.[39, 37, 2]). In the table below we show the present $K \to \pi\pi\gamma$ experimental status; also shown are the reason for the suppression of the bremsstrahlung amplitude and the leading multipole amplitudes.

decay	BR(bremsstrahlung)	BR(direct emission)
$K^\pm \to \pi^\pm\pi^0\gamma$ $T^*_{\pi^+} = (55-90)MeV$	$(2.57 \pm 0.16) \times 10^{-4}$ $(\Delta I = 3/2)$	$(4.72 \pm 0.77) \times 10^{-6}$ $E1, M1$
$K_S \to \pi^+\pi^-\gamma$ $E^*_\gamma > 50MeV$	$(1.78 \pm 0.05) \times 10^{-3}$	$< 6 \times 10^{-5}(E1)$
$K_L \to \pi^+\pi^-\gamma$ $E^*_\gamma > 20MeV$	$(1.49 \pm 0.08) \times 10^{-5}$ (CP violation)	$(3.09 \pm 0.06) \times 10^{-5}$ $M1, E2$
$K_S \to \pi^0\pi^0\gamma$		$M2$
$K_L \to \pi^0\pi^0\gamma$	$< 5.6 \times 10^{-6}$	$E2$

We do not discuss $K_{S,L} \to \pi^0\pi^0\gamma$ due to the small branching ratio ($< 10^{-8}$)[40]. $K_S \to \pi^+\pi^-\gamma$ has been discussed in [41] and no new experimental results have been reported recentely. While motivated by new results we update $K_L \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$.

$\underline{K_L \to \pi^+\pi^-\gamma(\gamma^*)}$: The bremsstrahlung ($E_B$) is suppressed by CP violation ($\sim \eta_{+-}$) and firmly established theoretically from (14) predicting $B(K_L \to \pi^+\pi^-\gamma)_{IB} = 1.42 \cdot 10^{-5}$ [42]. This contribution has been also measured by interference with the $M1$ transition in $K_L \to \pi^+\pi^-e^+e^-$ [43, 44]. Due to the large slope, KTeV parametrizes the magnetic amplitude in (12) as $e\mathcal{F}/M_K^4$ and

$$\mathcal{F} = \widetilde{g}_{M1}\left[\frac{a_1}{(M_\rho^2 - M_K^2) + 2M_K E^*_\gamma} + a_2\right] \qquad (16)$$

finding $a_1/a_2 = (-0.729 \pm 0.026(\text{stat}))GeV^2$ and the branching given in the table, which fixes also $\widetilde{g}_{M1}$. Such large slope can be accomodated in various Vector dominance schemes [40, 38, 45], while the rate is very sensitive to $SU(3)-$breaking and unknown p^4 unknown low energy contributions and thus difficult to predict.

$\underline{K^+ \to \pi^+\pi^0\gamma}$: New data from BNL E787[46] show vanishing interference among bremsstrahlung and electric transition. Thus the direct emission branching ($B(K^+ \to \pi^+\pi^0\gamma)^{DE}_{\text{exp}}$), in the table, must be interpreted as a pure magnetic transition. Theoretically one can identify two different sources for M, appearing at $\mathcal{O}(p^4)$: i) a pole diagram with a Wess-Zumino term and ii) a pure weak contact term, generated also in factorization by an anomalous current [47, 48]. $B(K^+ \to \pi^+\pi^0\gamma)^{DE}_{\text{exp}}$ is substantially smaller than previous values, but still show that the contribution ii) is non-vanishing.

3.1 CP Violation

Direct CP violation can be established in the width charge asymmetry in $K^\pm \to \pi^\pm\pi^0\gamma$, $\delta\Gamma/2\Gamma$ and in the interference E_B with E_1 in $K_L \to \pi^+\pi^-\gamma$ (E with M_1 in $K_L \to \pi^+\pi^-e^+e^-$);

both observables are kinematically difficult since one is looking for large photon energy distribution[2]. SM charge asymmetry were looked in [49] expecting $\delta\Gamma/2\Gamma \leq 10^{-5}$. General bounds on new physics in $M1$ transitions have been studied in [50], while the effects of dimension-5 operators in (2) $E1$ transitions have been studied in [51], where for instance it has been shown that the value of $\Re\left(\varepsilon'/\varepsilon\right)$ allows in particular kinematical regions a factor 10 larger than SM.

4 Conclusions

We think that the recent experimental results in K decays, for instance ε'/ε and $K^+ \to \pi^+ l^+ l^-$, encourage us to think that stringent tests of the SM and of its possible extensions are not too far ahead. From the theoretical side, radiative rare kaon decays can be a good laboratotory to understand very interesting questions like why the size of b_+/a_+ in (6) does not respect chiral counting. Interestingly a similar question for $K_L \to \pi^0\gamma\gamma$ got finally an answer, as we have shown in Section 2.3: the full $K \to 3\pi$ amplitude and VMD enhance the $\mathcal{O}(p^6)$ contributions. May be we have just to work harder for $K^+ \to \pi^+ l^+ l^-$, but at the end we may get predictive power and also interesting physics insight. For the people who think that theorists find always a good excuse I remind $K_S \to \gamma\gamma$ [52], where theory is very predictive and no large higher order contributions can be found. Finally interesting analytic approaches to weak matrix elements has been recently exploited in Ref.[53], where the relevant Green functions are evaluated and matched.

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